# SOLVING NONLINEAR HEAT AND MASS TRANSFER PROBLEMS WITH THE AID OF COMPUTERS

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A method is proposed of electrically simulating heat and mass transfer processes which can be described by a system of nonlinear equations, using for this purpose nonlinear resistances.

1. We first consider solving by electrical simulation the nonlinear heat conduction equations of the general form

$$\frac{\partial}{\partial x}\left(\lambda_q \frac{\partial t}{\partial x}\right) = c_q \frac{\partial t}{\partial \tau}, \tag{1}$$

with $\lambda = \lambda(t)$. Such problems can be solved, for example, by the Liebmann method with resistance networks. This method, where time derivatives are represented in discrete terms, is rather cumbersome.

The author proposes to solve the transfer problem with the aid of analog computers using nonlinear circuit elements, with voltage simulating the temperature and current simulating the thermal flux density. In the nonlinear heat transfer problem the temperature gradient and the thermal flux density are related through a proportionality factor which is a function of the temperature:

$$\mathbf{j}_q = -\left[\lambda_q(t)\right] \operatorname{grad} t. \tag{2}$$

This relation will be rewritten as a difference equation for the one-dimensional case:

$$\mathbf{j}_q \approx \frac{-\lambda_q(t_2)\, t_2 + \lambda_q(t_1)\, t_1}{\Delta x}.$$

Equation (1) becomes a difference equation:

$$\frac{\dfrac{\lambda_q}{c_q}(t_2)\, t_2 + \dfrac{\lambda_q}{c_q}(t_0)\, t_0 - 2\,\dfrac{\lambda_q}{c_q}(t_1)\, t_1}{(\Delta x)^2} = \frac{\partial t_1}{\partial \tau}. \tag{3}$$

Here the specific heat is also a function of the temperature.

For the system of nonlinear difference equations (3) one can construct the nonlinear model shown in Fig. 1, where the nonlinear resistors in blocks N. R-1 and N. R-2 simulate the nonlinear coefficients $c_q/\lambda_q(t)$. These resistor blocks are made up of diodes connected in parallel, with both their bias voltages and the slopes of their volt–ampere characteristics adjustable over wide ranges so as to ensure the feasibility of piecewise-linearly approximating the nonlinear curve

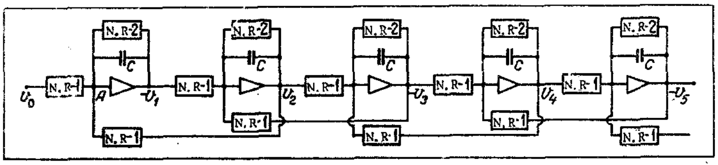


Fig. 1. Schematic diagram of the electrical model for solving a nonlinear heat transfer problem.

K. A. Timiryazev Agricultural Academy, Moscow. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 22, No. 3, pp. 548-552, March 1972. Original article submitted March 24, 1971.

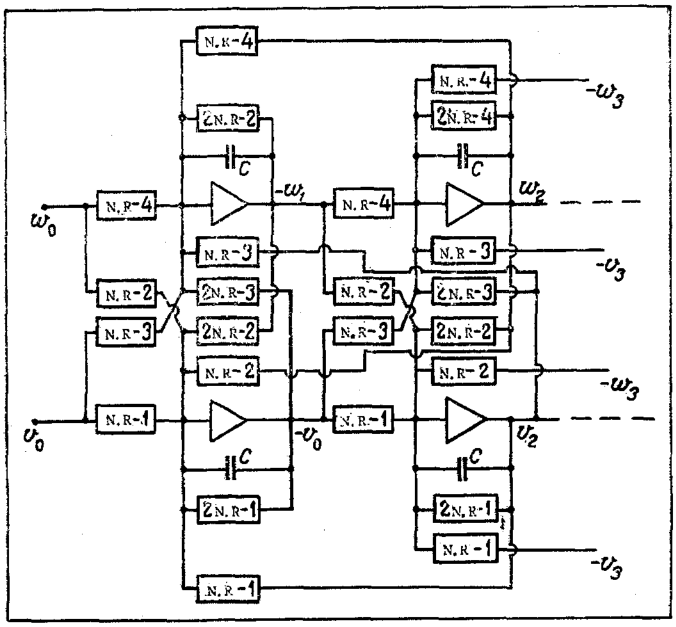


Fig. 2. Schematic diagram of the electrical model for solving a nonlinear heat and mass transfer problem.

$$\frac{\lambda_q}{c_q}(t)\,t. \tag{4}$$

The corresponding relation on the model is

$$i = \frac{1}{R(v)}v,$$

where R(v) is the nonlinear resistance of an N.R block [5, 6].

Into a certain nodal point A (Fig. 1) flow the following currents:

$$i_0 = \frac{v_0}{R(v_0)};\quad i_2 = \frac{v_2}{R(v_2)};\quad i_1 = -\frac{v_1}{0{,}5R(v_1)};\quad i_c = -\frac{dv_1}{d\tau_e}$$

(the capacitance in an integrating circuit is taken as equal to unity) and, as a result, the following equation is simulated:

$$\frac{v_0}{R(v_0)} + \frac{v_2}{R(v_2)} - \frac{v_1\cdot 2}{R(v_1)} - \frac{dv_1}{d\tau_e} = 0; \tag{5}$$

the latter being an analog of the difference equation (3).

A major advantage of the proposed model is that all nonlinear N.R-1 blocks, which simulate nonlinear thermal resistances $c_q/\lambda_q(t)$, are the same; the N.R-2 blocks are also all the same, but differ from the N.R-1 blocks by the factor 2: this follows from Eq. (5), where the third term appears with the coefficient 2.

It is well known that the nonlinear circuits of analog computers contain up to 20 adjustable diode elements, i.e., that the nonlinear relation between current and voltage can be approximated by a broken straight line of up to 20 segments, while the necessary accuracy can be obtained almost entirely with not more than 3-4 segments. A further convenience offered by this model is that, unlike in the Liebmann method, it is set up once and for all with identical components and the resistors do not have to be changed during a simulation process; rather, the voltage is varied continuously, which contributes to the stability and the convergence of the simulation process.

2. We will now consider solving by electrical simulation the nonlinear heat and mass transfer equation in [1]:

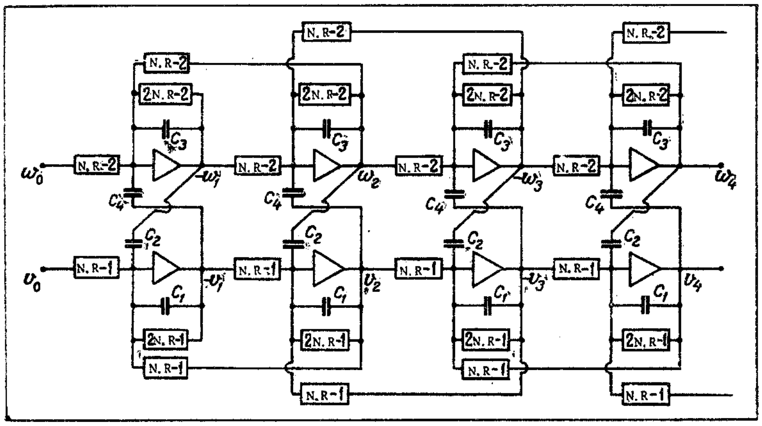


Fig. 3. Schematic diagram for solving the special heat and mass transfer problem.

$$\begin{cases} c_q \dfrac{\partial t}{\partial \tau} = \dfrac{\partial}{\partial x}\left(\lambda_q \dfrac{\partial t}{\partial x}\right) + \varepsilon r \dfrac{\partial \theta}{\partial \tau}, \\ \dfrac{\partial \theta}{\partial \tau} = \dfrac{\partial}{\partial x}\left(a_m \dfrac{\partial \theta}{\partial x}\right) + \dfrac{\partial}{\partial x}\left(a_m \delta_\theta \dfrac{\partial t}{\partial x}\right). \end{cases} \tag{6}$$

A method of electrically simulating such a system of nonlinear equations by means of resistances has been proposed by L. A. Kozdoba in [2] and, for a more general case, in [3].

We propose here to simulate system (6) by means of amplifiers and nonlinear resistors.

As in the preceding case, we approximate Eqs. (6) by difference equations

$$\begin{aligned} \frac{\partial t_{i+1}}{\partial \tau} &= \left(\lambda_q + \frac{c_m}{c_q} a_m\right)_{i+2} t_{i+2} + \left(\lambda_q + \frac{c_m}{c_q} a_m\right)_i t_i \\ &\quad - 2\left(\lambda_q + \frac{c_m}{c_q} a_m\right)_{i+1} t_{i+1} + \left(\frac{\varepsilon r}{c_q}\lambda_m\right)_{i+2} \theta_{i+2} \\ &\quad + \left(\frac{\varepsilon r}{c_q}\lambda_m\right)_i \theta_i - 2\left(\frac{\varepsilon r}{c_q}\lambda_m\right)_{i+1} \theta_{i+1}, \\ \frac{\partial \theta_{i+1}}{\partial \tau} &= (a_m\theta)_{i+2} + (a_m\theta)_i - 2(a_m\theta)_{i+1} \\ &\quad + (a_m\delta_\theta t)_{i+2} + (a_m\delta_\theta t)_i - 2(a_m\delta_\theta t)_{i+1}. \end{aligned} \tag{7}$$

These difference equations can be simulated on analog computers by two amplifier networks with nonlinear resistors. Two stages of this model are shown in Fig. 2.

Let each coefficient of the unknown functions in Eqs. (7) depend on one of the unknown functions only, namely the function with which it is associated.

The N. R-1 blocks simulate the nonlinear dependence of

$$\left(\lambda_q + \frac{c_m}{c_q} a_m\right)$$

on the temperature t, the N. R-2 blocks simulate the dependence of $\varepsilon r(\lambda_m/c_q)$ on $\theta$; the N. R-3 blocks simulate the dependence of $a_m\delta_\theta$ on the temperature t, and the N. R-4 blocks simulate the dependence of $a_m$ on $\theta$.

Doubling the N. R-3 resistance is indicated on the diagram by 2N. R-3.

Similar designations indicate here the doubling of other resistances.

3. The process of heat and mass transfer is particularly strongly affected by the nonlinear dependence of $\lambda_q$ on t and $a_m$ on $\theta$. In view of this, we will consider the problem with $\delta_\theta$ = const. and $\varepsilon$ = const., i.e., with the Pn number and the Ko* number both constant, while $\lambda = \lambda(t)$ and $a_m = a_m(\theta)$ are given.

Then Eqs. (6) in dimensionless form become [1]:

$$\begin{cases} \dfrac{\partial T}{\partial \mathrm{Fo}} = \dfrac{\partial}{\partial X}\left(\dfrac{\lambda}{\lambda^*} \cdot \dfrac{\partial T}{\partial X}\right) + \mathrm{Ko}^* \dfrac{\partial \Theta}{\partial \mathrm{Fo}}, \\ \dfrac{\partial \Theta}{\partial \mathrm{Fo}} [1 + \mathrm{Lu\,Pn\,Ko}^*]_{\mathrm{mean}} = \dfrac{\partial}{\partial X}\left(\mathrm{Lu} \dfrac{\partial \Theta}{\partial X}\right) + (\mathrm{Lu\,Pn})_{\mathrm{mean}} \dfrac{\partial T}{\partial \mathrm{Fo}}. \end{cases} \tag{8}$$

Here the coefficients of $\partial\Theta/\partial \mathrm{Fo}$ and $\partial T/\partial \mathrm{Fo}$ refer to some mean values of the thermophysical properties in the given process. The Luikov number ($\mathrm{Lu} = a_m/a_q^*$) is a nonlinear function of $\Theta$, with some constant (mean) quantity $a_q^*$ in the denominator.

The ratio $\lambda_q/\lambda_q^*$ is also a nonlinear function of T.

As in the preceding case, Eqs. (8) will be converted to difference equations and a model will be constructed as shown in Fig. 3. Here the capacitances $C_1$, $C_2$, $C_3$, and $C_4$ are already different, respectively proportional to 1, $\mathrm{Ko}^* (1 + \mathrm{Pn\,Ko}^* \mathrm{Lu})_{\mathrm{mean}}$, and $(\mathrm{PnLu})_{\mathrm{mean}}$.

## NOTATION

$t$ is the temperature;
$\theta$ is the mass-transfer potential;
$\tau$ is the time;
$\lambda_q$ is the thermal conductivity;
$c_q$ is the specific heat;
$a_m$ is the mass diffusivity;
$\varepsilon$ is the ratio of mass change by phase transformation to total mass change;
$r$ is the specific heat of phase transformation;
$\delta_\theta$ is the temperature gradient coefficient;
$j_q$ is the vector of thermal flux density;
$T$ is the dimensionless temperature;
$\Theta$ is the dimensionless mass transfer potential;
Fo is the Fourier number;
Lu is the Luikov number;
Ko* is the Kossovich number;
Pn is the Posnov number;
v and w is the electric potential simulating t and $\theta$, respectively;
R(v) is the nonlinear electrical resistance, a function of v;
$\tau_e$ is the machine time (on the electrical model);
$\lambda_q^*$, $a_m^*$ are the characteristic thermophysical constants.

## LITERATURE CITED


1. A. V. Lykov and Yu. A. Mikhailov, Theory of Heat and Mass Transfer [in Russian], Gosenergoizdat (1963).
2. L. A Kozdoba, Inzh.-Fiz. Zh., 6, No. 5 (1963).
3. L. A. Kozdoba and V. A. Zagoruiko, Inzh.-Fiz. Zh., 11, No. 5 (1966).
4. G. Liebmann, "A new electrical analog method for the solution of transient heat conduction problems," Trans. ASME, 78 (1956).
5. U. Carplus, Simulating Devices for Solving Field Theory Problems [Russian translation], Inostr. Lit. (1962).
6. B. Ya. Kogan, Electronic Simulating Devices and Their Application to the Analysis of Automatic Regulation Systems [in Russian], Fizmatgiz (1963).